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$D = 4$, $\mathcal{N} = 2$ Gauged Supergravity in the Presence of Tensor Multiplets.

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ABSTRACT

Using superspace techniques we construct the general theory describing $D = 4$, $\mathcal{N} = 2$ supergravity coupled to an arbitrary number of vector and scalar–tensor multiplets. The scalar manifold of the theory is the direct product of a special Kähler and a reduction of a Quaternionic–Kähler manifold. We perform the electric gauging of a subgroup of the isometries of such manifold as well as "magnetic" deformations of the theory discussing the consistency conditions arising in this process. The resulting scalar potential is the sum of a symplectic invariant part (which in some instances can be recast into the standard form of the gauged $\mathcal{N} = 2$ theory) and of a non–invariant part, both giving new deformations. We also show the relation of such theories to flux compactifications of type II string theories.

1   Introduction

Compactifications of type II string theory on Calabi–Yau manifolds provide effective four–dimensional theories which can be described by $\mathcal{N} = 2$ supergravity coupled to matter. The theories obtained in this way naturally contain tensor multiplets, but, using the known duality relation between tensor and scalar fields in four dimensions, one can use the standard formulation in terms of hypermultiplets. On the other hand, if in addition to the metric one gives non–trivial expectation values to the other fields of the ten–dimensional theory, the effective supergravity in four dimensions is deformed and various fields become massive. Among these there are the tensors which therefore cannot be dualized into scalars anymore.

These type of deformations are usually described by gauged supergravity theories and the general couplings for the $d = 4$, $\mathcal{N} = 2$ case have been worked out in [1, 2]. However, the description provided in [1, 2] does not include (massive) tensor multiplets and it is therefore difficult to establish its relation with flux compactifications of type II strings. Moreover, it is known that the gauging procedure may give inequivalent deformations of theories which can be mapped onto each other before the gauging is performed. For these reasons it is quite important to build the general theory describing four–dimensional $\mathcal{N} = 2$ gauged supergravity coupled to tensor multiplets and possibly to establish the relation of such a formulation with the standard one of [2].

Although effective theories of type II compactifications with fluxes have been described in [3, 4, 5, 6, 7, 8, 9] and especially [7] shows the explicit appearance of tensor fields, we provide here the general construction of $\mathcal{N} = 2$ supergravity with tensor multiplets and describe its gauging also finding new deformations. The starting point for our construction[1] is given by [11], where the tensor multiplet couplings to supergravity were described and some of the relations on the scalar geometry of the theory were given.

In this paper we improve the results of [11] by extending the couplings to vector multiplets and by performing the gauging of the theory. More in detail, by using superspace techniques we get the general supersymmetry rules after the dualization of some of the scalars of the hypermultiplets into tensor fields. As a first step in the construction, we discuss the necessary conditions that have to be satisfied in order to perform such a dualization. We then describe the constraints on the resulting geometry and show their relations with the underlying quaternionic geometry of the hypermultiplet scalar σ–model. In particular, we will show how these constraints can be understood from a "Kaluza–Klein" perspective. When the quaternionic manifold is given by a homogeneous space, the dualization procedure yields a space which can be described as the reduction of the original one by removing some of its nilpotent generators under the solvable decomposition.

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[1]For earlier work see [10].

As a next step we perform the gauging of the theory. After dualization, not all of the isometries of the original manifold remain isometries of the final scalar manifold. Moreover, some of these act non–trivially on the tensor fields and therefore cannot become local symmetries without leading to non–linear couplings for the tensor fields. We then discuss which isometries can be made local and therefore "gauged". Always using the superspace formalism we compute the fermion shifts which restore the supersymmetry of the theory and give rise to a potential satisfying the supersymmetry Ward identities.

The appearance of tensor fields allows to redefine the gauge field strengths with a shift proportional to the tensor fields $F^\Lambda \to F^\Lambda + m^{I\Lambda} B_I$ without breaking supersymmetry, provided we redefine appropriately the fermion transformation laws. Here $m^{I\Lambda}$ are real constants which can be thought as mass parameters for the tensors. This kind of extension[2] of the theory was first obtained in [13] for six–dimensional supergravity, further extended in [14] and shown in Calabi–Yau compactification of Type II theories in [7].

Indeed the gauging we perform after dualization of some of the hypermultiplets scalars is a standard electric gauging, but the appearance of the mass parameters $m^{I\Lambda}$ in the definition of the new gauge field strengths implies the existence of extended solutions. The shifts of the supersymmetry transformations indeed acquire some extra terms depending on such parameters so that the gravitino's and hyperino's shifts are symplectic invariants. This latter can be interpreted also as a "magnetic" gauging, though its definition is not related to the appearance of magnetic gauge fields. These would lead to the construction of [4] whose consistency is problematic, as explained in [6, 7].

The scalar potential of the theory follows as usual from the square of the fermionic shifts by using a known Ward identity of N–extended gauged supergravities [15]. Being the square of symplectic invariant quantities, but for a term coming from the gaugino shift when non–Abelian isometries of the Special Kähler manifold are gauged, the potential shows symplectic invariance for Abelian gaugings where such gaugino contribution does not appear. Therefore the potential can be split in two parts, one that is explicitly symplectic invariant while the other is not. The first part is in particular the one which can be obtained by gaugings of translational isometries and it is therefore directly related to the one which follows from flux compactifications [7]. However we can not reduce by a symplectic transformation this part to the standard one described in [2], unless we have a single tensor or in the case where all the symplectic vectors of the electric and magnetic charges are parallel vectors. Therefore, except for this particular situation, the magnetic charges give a genuine deformation of the theory with respect to the standard $\mathcal{N} = 2$ supergravity. This is a fortiori true for non

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[2]Tensor multiplets exist also in 5 dimensions and they can get masses by the same procedure. However, since they satisfy first order equations of motion, the gauging of the theory can give additional couplings with the vectors [12].

abelian gaugings, since a non symplectic invariant extra term is present.

It is interesting to point out that in this process we also get new consistency conditions with respect to those of the underlying quaternionic geometry. An especially interesting condition is given by the requirement that a certain combination of the mass parameters must belong to the center of the gauged Lie Algebra. This condition is certainly satisfied in our case because each term of the linear combination is in the center of the algebra since we assume that the isometries that can be gauged electrically are only those which commute with the translational isometries of the dualized scalars. As we will show, these mass parameters can be interpreted as "magnetic" Killing vectors and therefore this condition has a natural interpretation as the fact that the electric and magnetic generators should commute.

The plan of the paper is the following. After this introduction, in section 2 we review the dualization procedure in order to fix notations, discuss the geometry of the scalar manifold and give the couplings of $\mathcal{N} = 2$ supergravity to vector, tensor and hypermultiplets. In section 3 we discuss in detail the gauging procedure. By solving the Bianchi identities of the various fields using superspace techniques we obtain the shifts to the supersymmetry transformations and determine the potential of the coupled theory. We then discuss the properties of such a potential in section 4 detailing the relations with flux compactifications and with the standard theory of [2]. We also give an Appendix where some explicit examples of dualizations are discussed.

2 Tensor multiplets coupled to supergravity and vector multiplets

As already explained in the introduction, the standard $\mathcal{N} = 2$ supergravity contains both vector multiplets and hypermultiplets. The σ–model parametrized by the scalars of the vector multiplets is a special Kähler manifold while for the hypermultiplets the σ–model is quaternionic–Kähler. When we dualize some of the scalars of the quaternionic manifold to tensor fields, the geometry of the hypermultiplets sector is modified and, in particular, it is no more quaternionic. As a first step in our construction we will therefore review the dualization procedure so that we can also fix our notations.

Before entering the details of the dualization procedure some comments are in order. Quaternionic–Kähler manifolds do not necessarily admit isometries. On the other hand, the standard dualization of a hypermultiplet scalar field q into a tensor $B_{\mu\nu}$ requires that q appears in the Lagrangian only through its derivatives so that the Lagrangian is invariant under constant shifts

$$q \to q + \eta \,. \tag{2.1}$$

Moreover, if one wants to obtain more than one tensor field by this procedure, the isometries associated to the dualized scalars should commute, so that all of them can be described by equation (2.1) at the same time. Notice that so far we did not make any distinction between compact and non–compact isometries, though as it is shown in the third example of the appendix, the resulting physics is very different, actually we get in general a singular Lagrangian.

2.1 Dualization of the commuting isometries

Dualization of the commuting isometries in the quaternionic manifold spanned by the hypermultiplets can be done in the usual way by a Legendre transformation on the quaternionic coordinates $q^{\hat{u}}$, $\hat{u} = 1, \ldots, 4m$ appearing in the Lagrangian covered by derivatives. Here and in the following, we are using for the quaternionic geometry the notations given in [2, 15].

Suppose we partition the coordinates $q^{\hat{u}}$, $\hat{u} = 1, \ldots, 4m$ into the two subsets q^u, $u = 1, \ldots, n$ and q^I, $I = 1, \ldots, 4m - n$, where q^I are the coordinates we want to dualize. Since we are interested in the geometry of the resulting manifold $\mathcal{M}_n$ we just consider the dualization of the quaternionic kinetic term $\mathcal{L}_{\mathcal{K}}$ in the general $\mathcal{N} = 2$ Lagrangian. If dq^I is considered as an independent 1-form field, $dq^I = \Phi^I$ and if we add a 3–form Lagrangian multiplier $H_I = dB_I$ to $\mathcal{L}_{\mathcal{K}}$, we have:

$$\mathcal{L}_{\mathcal{K}} = -h_{\hat{u}\hat{v}}\, dq^{\hat{u}} \wedge *dq^{\hat{v}} \Rightarrow \tag{2.2}$$
$$-h_{uv}\, dq^u \wedge *dq^v - 2h_{uI}\, dq^u \wedge *\Phi^I - h_{IJ}\, \Phi^I \wedge *\Phi^J + \Phi^I \wedge H_I\,.$$

Varying $\mathcal{L}_{\mathcal{K}}$ with respect to B_I we find that Φ^I is closed and therefore one can equate $\Phi^I = dq^I$. The variation with respect to Φ^I gives

$$* \Phi^I = M^{IJ} \left(\frac{1}{2} H^J - h_{uJ} * dq^u \right) \tag{2.3}$$

and substituting this in (2.2) we find

$$\mathcal{L}_{\mathcal{K}}^{(dual)} = -g_{uv} dq^u \wedge *dq^v + \frac{1}{4} M^{IJ} H_I \wedge H_J - M^{IJ} h_{uI}\, dq^u \wedge H_J\,, \tag{2.4}$$

where

$$g_{uv} = h_{uv} - M_{IJ} A_u^I A_v^J\,, \quad M_{IJ} \equiv h_{IJ}\,, \quad M_{IJ} A_u^J = h_{Iu}\,, \tag{2.5}$$

and we have defined $M_{IJ} M^{JK} = \delta_I^K$.

As already observed in [11] the metric for the residual scalars, the metric for the kinetic term of the tensors and for their couplings to the scalars, correspond to the Kaluza–Klein decomposition of the quaternionic metric

$$h_{\hat{u}\hat{v}} = \epsilon_{AB}\, \mathbb{C}_{\alpha\beta}\, \mathcal{U}_{\hat{u}}^{A\alpha}\, \mathcal{U}_{\hat{v}}^{B\beta} \tag{2.6}$$

that is

$$h_{\hat{u}\hat{v}} = \begin{pmatrix} g_{uv} + M_{IJ}A_u^I A_v^J & M_{JK}A_u^K \\ M_{IK}A_v^K & M_{IJ} \end{pmatrix}, \tag{2.7}$$

$$h^{\hat{u}\hat{v}} = \begin{pmatrix} g^{uv} & -g^{uw}A_w^J \\ -g^{vw}A_w^I & M^{IJ} + g^{uv}A_u^I A_v^J \end{pmatrix}. \tag{2.8}$$

The analogous dualization of $\mathcal{L}_{\mathcal{K}}$ made in terms of the quaternionic vielbein $\mathcal{U}_{\hat{u}}^{A\alpha}$ gives

$$\mathcal{U}_u^{A\alpha} = P_u^{A\alpha} + A_u^I \mathcal{U}_I^{A\alpha}, \tag{2.9}$$

with

$$P_u^{A\alpha} P_u^{B\beta}\, \mathbb{C}_{\alpha\beta}\, \epsilon_{AB} = g_{uv}. \tag{2.10}$$

The fact that the reduction of the quaternionic metric has the same structure as a Kaluza–Klein reduction on the torus was to be expected since in both cases we consider isometries described by constant Killing vectors. However, while in the ordinary Kaluza–Klein the Killing vectors are generators of compact isometries, here we consider only Killing vectors associated to non compact isometries, namely translations. Indeed if we perform a dualization of a compact coordinate covered by the derivative in a generic σ-model then, as shown in the example 3 of the appendix, one usually obtains a singular Lagrangian, while the dualization of non-compact coordinates gives a regular Lagrangian. The geometrical procedure associated to such dualization can be easily described in the case of quaternionic σ-models which are symmetric spaces G/H. Indeed let us perform the solvable decomposition of the Lie algebra G as

$$\mathfrak{g} = \mathfrak{h} \oplus Solv(\mathfrak{g}) = \mathfrak{h} \oplus \mathbb{D} \oplus \mathbb{N}, \tag{2.11}$$

where $\mathbb{D}$ is the non-compact Cartan subalgebra and $\mathbb{N}$ denote the nilpotent part of the algebra. We consider in $\mathbb{N}$ the maximal abelian subalgebra of maximal dimension which can be generated by translational Killing vectors. Considering any subset of such Killing vectors and deleting the corresponding generators in $\mathfrak{g}$ corresponds to dualizing the isometries associated to the coordinates q^I. In the appendix we give two examples of such procedure for the quaternionic manifolds $SU(2,1)/SU(2) \times U(1)$ and $SO(1,4)/SO(4)$. Geometrically this corresponds to considering the quaternionic manifold as a fiber bundle whose base space is $\mathcal{M}_n$ and fiber given by the set of coordinates corresponding to the commuting Killing vectors. The projections along the fibers correspond to our dualization procedure.

2.2 Reduction of the quaternionic geometry after dualization

It is interesting to reduce the coordinate indices of the quaternionic geometry for some important formulae. Let us first consider the quaternionic relation[3]

$$(\mathcal{U}_{\hat{u}}^{A\alpha}\mathcal{U}_{\hat{v}}^{B\beta} + \mathcal{U}_{\hat{v}}^{A\alpha}\mathcal{U}_{\hat{u}}^{B\beta})\mathbb{C}_{\alpha\beta} = h_{\hat{u}\hat{v}}\epsilon^{AB} \,. \tag{2.12}$$

where ϵ^{AB} and $\mathbb{C}_{\alpha\beta}$ are the usual antisymmetric metrics used to raise and lower indices of SU(2) and Sp(2m). Splitting (2.12) on the uv , uI , IJ indices we find three equations which imply

$$(P_u^{A\alpha}P_v^{B\beta} + P_v^{A\alpha}P_u^{B\beta})\mathbb{C}_{\alpha\beta} = g_{uv}\epsilon^{AB} \,, \tag{2.13}$$

$$(\mathcal{U}_I^{A\alpha}\mathcal{U}_J^{B\beta} + \mathcal{U}_J^{A\alpha}\mathcal{U}_I^{B\beta})\mathbb{C}_{\alpha\beta} = M_{IJ}\epsilon^{AB} \,, \tag{2.14}$$

$$P_u^{A\alpha}\mathcal{U}_{B\alpha I} + \mathcal{U}_I^{A\alpha}P_{B\beta u} = 0 \,. \tag{2.15}$$

Furthermore, using (2.8) we can easily obtain

$$\mathcal{U}^{A\alpha u} = g^{uv}P_v^{A\alpha} \equiv P^{uA\alpha} \,, \tag{2.16}$$

$$\mathcal{U}^{A\alpha I} = M^{IJ}\mathcal{U}_J^{A\alpha} - g^{vw}A_w^I P_v^{A\alpha} \,. \tag{2.17}$$

Further insight into the geometrical structure of the sigma model $\mathcal{M}_n$ is obtained by reduction of the quaternionic indices $\hat{u}$, $\hat{v}$ for the connections and curvatures of the holonomy group contained in $\mathrm{Sp}(2m) \times \mathrm{SU}(2)$. The SU(2) curvature 2–form is defined as

$$\hat{\Omega}^x = d\hat{\omega}^x + \frac{1}{2}\epsilon^{xyz}\hat{\omega}^y \wedge \hat{\omega}^z \,, \tag{2.18}$$

where $\hat{\omega}^x$ is the SU(2) connection. Defining

$$\hat{\omega}^{AB} = \frac{\mathrm{i}}{2}\sigma_x^{AB}\hat{\omega}^x \,, \quad \hat{\omega}^x = -\mathrm{i}\,\hat{\omega}^{AB}\sigma_{AB}^x \,, \tag{2.19}$$

where σ_x^{AB} are symmetric matrices related to the Pauli matrices

$$\sigma_x^{AB} \equiv \epsilon^{CA}\sigma_{x\,C}^{\ \ B} \,, \tag{2.20}$$

one obtains

$$\hat{\Omega}^{AB} \equiv d\hat{\omega}^{AB} + \hat{\omega}^{(AC} \wedge \hat{\omega}_C^{\ B)} \,. \tag{2.21}$$

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[3]Throughout the paper we use hats to denote quantities referring to the original quaternionic–Kähler manifold and the same symbols without hats for the reduced one $\mathcal{M}_n$.

Reducing the coordinates indices we find for the connections

$$\hat{\omega}_u^{AB} = \hat{\omega}_{C\alpha}^{AB}\mathcal{U}_u^{C\alpha} = \hat{\omega}_{C\alpha}^{AB}\left(P_u^{C\alpha} + A_u^I\mathcal{U}_I^{C\alpha}\right) \equiv \omega_u^{AB} + A_u^I\omega_I^{AB}\,, \tag{2.22}$$

$$\hat{\omega}_I^{AB} = \omega_I^{AB}\,, \tag{2.23}$$

$$\hat{\Delta}_u^{\alpha\beta} = \hat{\Delta}_{C\alpha}^{\alpha\beta}\mathcal{U}_u^{C\alpha} = \hat{\Delta}_{C\alpha}^{AB}\left(P_u^{C\alpha} + A_u^I\mathcal{U}_I^{C\alpha}\right) \equiv \Delta_u^{\alpha\beta} + A_u^I\Delta_I^{\alpha\beta}\,, \tag{2.24}$$

$$\hat{\Delta}_I^{\alpha\beta} = \Delta_I^{\alpha\beta}\,. \tag{2.25}$$

It is also important to reduce the SU(2) curvature and its relation to the quaternionic pre-potential. Using equations (2.22), (2.23) and the fact that after dualization all the quantities depend just on the q^u so that the derivative ∂_I is always zero, one obtains from (2.21)

$$\hat{\Omega}_{uv}^{AB} = \Omega_{uv}^{AB} + F_{uv}^I\omega_I^{AB} + 2A_{[v}^I\Omega_{u]I}^{AB} + A_u^I A_v^J\Omega_{IJ}^{AB}\,, \tag{2.26}$$

$$\hat{\Omega}_{uI}^{AB} = \Omega_{uI}^{AB} + A_u^J\Omega_{JI}^{AB}\,, \tag{2.27}$$

$$\hat{\Omega}_{IJ}^{AB} = \Omega_{IJ}^{AB}\,, \tag{2.28}$$

where the components of the reduced Lie algebra valued SU(2) curvatures are defined as usual:

$$\Omega_{uv}^{AB} \equiv \partial_{[u}\omega_{v]}^{AB} + \omega_{[u}^{AC}\omega_{v]C}{}^B\,, \tag{2.29}$$

$$\Omega_{uI}^{AB} \equiv \partial_{[u}\omega_{I]}^{AB} + \omega_{[u}^{AC}\omega_{I]C}{}^B = \frac{1}{2}\nabla_u\omega_I^{AB}\,, \tag{2.30}$$

$$\Omega_{IJ}^{AB} \equiv \partial_{[I}\omega_{J]}^{AB} + \omega_{[I}^{AC}\omega_{J]C}{}^B = \omega_{[I}^{AC}\omega_{J]C}{}^B\,. \tag{2.31}$$

On the other hand, for the quaternionic–Kähler geometry the SU(2) curvature $\hat{\Omega}^{AB}$ is defined in terms of the quaternionic vielbeins as

$$\hat{\Omega}^{AB} = -\mathcal{U}^A{}_\alpha \wedge \mathcal{U}^{B\beta}\,, \tag{2.32}$$

and therefore reducing the r.h.s. we can also get

$$\hat{\Omega}_{uv}^{AB} = -\mathcal{U}_{[u\,\alpha}^A\mathcal{U}_{v]}^{B\alpha} = -P_{[u\,\alpha}^A P_{v]}^{B\alpha} - A_{[u}^I A_{v]}^J\mathcal{U}_{[I\,\alpha}^A\mathcal{U}_{J]}^{B\alpha}\,, \tag{2.33}$$

$$\hat{\Omega}_{uI}^{AB} = -\frac{1}{2}\left(\mathcal{U}_{u\,\alpha}^A\mathcal{U}_I^{B\alpha} - \mathcal{U}_{I\,\alpha}^A\mathcal{U}_u^{B\alpha}\right) = -P_{u\,\alpha}^{(A}\mathcal{U}_I^{B)\alpha} + A_u^J\Omega_{JI}^{AB}\,, \tag{2.34}$$

$$\hat{\Omega}_{IJ}^{AB} = -\mathcal{U}_{[I\,\alpha}^A\mathcal{U}_{J]}^{B\alpha}\,. \tag{2.35}$$

From equations (2.26)–(2.28), (2.29)–(2.31), (2.33)–(2.35) we readily obtain

$$\Omega_{uv}^{AB} = -\nabla_{[u}(A_{v]}^I\omega_I^{AB}) - P_{\alpha[u}^A P_{v]}^{B\alpha}\,, \tag{2.36}$$

$$\nabla_u\omega_I^{AB} = -2P_{u\alpha}^{(A}\mathcal{U}_I^{B)\alpha}\,, \tag{2.37}$$

$$\omega_{[I}^{AC}\omega_{J]C}{}^B = -\mathcal{U}_{\alpha[I}^A\mathcal{U}_{J]}^{B\alpha}\,. \tag{2.38}$$

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Furthermore, from the reduction of the quaternionic torsion equation

$$\hat{\nabla}_{[\hat{u}}\mathcal{U}_{\hat{v}]}^{A\alpha} = 0 \tag{2.39}$$

we obtain the following relations

$$\nabla_{[u}P_{v]}^{A\alpha} = -F_{uv}^I\mathcal{U}_I^{A\alpha}, \qquad F_{uv}^I \equiv \partial_{[u}A_{v]}^I, \tag{2.40}$$

$$\nabla_u\mathcal{U}_I^{A\alpha} = \omega_{IB}^A P_u^{B\alpha} + \Delta_{I\beta}^\alpha P_u^{A\beta}, \tag{2.41}$$

$$\omega_{B[I}^A\mathcal{U}_{J]}^{B\alpha} = 0. \tag{2.42}$$

Note that in equation (2.40) there appears a "torsion" term F_{uv}^I; nevertheless this term is not related to the torsion of the connection on the reduced scalar manifold. In fact, from the torsionless equation of the original quaternionic manifold (2.39) we see that the covariant derivative acting on the quaternionic vielbein can be split as

$$\hat{\nabla}_u\mathcal{U}_v^{A\alpha} \equiv \partial_u\mathcal{U}_v^{A\alpha} - \hat{\Gamma}_{uv}^w\mathcal{U}_w^{A\alpha} - \hat{\Gamma}_{uv}^I\mathcal{U}_I^{A\alpha} + \hat{\omega}_{u}{}^A{}_B\mathcal{U}_v^{B\alpha} + \hat{\Delta}_u{}^\alpha{}_\beta\mathcal{U}_v^{A\beta}, \tag{2.43}$$

where the quantities appearing in (2.43) were defined in equations (2.22), (2.24), (2.9) and $\hat{\Gamma}$ are the components of the Levi–Civita connection associated to the quaternionic metric $h_{\hat{u}\hat{v}}$. Considering the antisymmetrizations on the indices u, v, the contribution of the Levi–Civita connection vanishes and substituting the expressions (2.22), (2.24), (2.9) one obtains equation (2.40). It is now evident that the torsion term F_{uv}^I arising in the reduction of the quaternionic–Kähler geometry is associated to the torsion of the connection Γ_{uv}^I which is not the connection on the scalar manifold $\mathcal{M}_n$. Indeed from the statement that the quaternionic metric is covariantly constant

$$\hat{\nabla}_{\hat{u}}h_{\hat{v}\hat{w}} = 0 \tag{2.44}$$

one can deduce that also the metric on the reduced scalar manifold $\mathcal{M}_n$ is covariantly constant

$$\nabla_u g_{vw} = 0 \tag{2.45}$$

where ∇_u is the covariant derivative with respect to the the Levi–Civita connection on the reduced scalar manifold with metric g_{uv}.

On the other hand, one can observe that the rectangular matrix $P_u^{A\alpha}$ is not the vielbein on the reduced manifold. Such vielbein 1–form, which we may denote E^i ($i = 1, \ldots n$), must satisfy $\nabla E^i = 0$ where ∇ contains the spin connection of $\mathcal{M}_n$, in accordance to the fact that the manifold is torsionless, according to equation (2.45).

From equation (2.40) it is also possible to evaluate the Riemannian curvature on the reduced scalar manifold acting with a further covariant derivative; one obtains:

$$\mathcal{R}_{vzwu} = -\Omega_{uv}^{AB}P_{zA\alpha}P_{vB}^\alpha - \mathbb{R}_{wu}{}^\alpha{}_\beta P_{zA\alpha}P_v^{A\beta} - M_{IJ}F_{v[u}^I F_{w]z}^J. \tag{2.46}$$

which shows explicitly, as already observed in reference [11], that the holonomy of the reduced scalar manifold is not contained in $\mathrm{SU}(2) \times \mathrm{Sp}(2m)$.

Finally, for the interpretation of the gauged supergravity theory coupled to tensor multiplets, it is important to reduce the fundamental relation defining the quaternionic prepotential (see for instance [2, 15]), namely

$$2k_\Lambda^{\hat{u}} \hat{\Omega}_{\hat{u}\hat{v}}^x = -\nabla_{\hat{v}} \mathcal{P}_\Lambda^x \,. \tag{2.47}$$

Writing equation (2.47) with explicit free index $\hat{v} \to (v, I)$ and setting $k_\Lambda^I = 0$ we get two equations which combined give the following relations

$$2k_\Lambda^u (\Omega_{uv}^x + F_{uv}^I \omega_I^x) - k_\Lambda^u A_u^I \nabla_v \omega_I^x = -\nabla_v \mathcal{P}_\Lambda^x \,, \tag{2.48}$$

$$k_\Lambda^u (\nabla_u \omega_I^x + 2A_u^J \Omega_{JI}^x) + \epsilon^{xyz} \omega_I^y \mathcal{P}_\Lambda^z = 0 \,. \tag{2.49}$$

2.3 Coupling of the scalar–tensor multiplet to $\mathcal{N} = 2$ ungauged supergravity with vector multiplets

After the dualization of some of the hypermultiplets, the original quaternionic–Kähler manifold reduces to a scalar manifold $\mathcal{M}_n$ whose scalar fields are part of the so called scalar–tensor multiplet

$$\{B_{I\,\mu\nu}, \zeta^\alpha, \zeta_\alpha, q^u\} \,, \tag{2.50}$$

where the index I takes n_T values, n_T being the number of coordinates covered by derivative which have been dualized, $\alpha = 1, \ldots m$ and $u = 1, \ldots n$, $n = 4m - n_T$.

We recall [2] that the content of the gravitational and vector multiplets is

$$\{V_\mu^a, A_\mu^0, \psi_A, \psi^A\} \,, \tag{2.51}$$

$$\{A_\mu^i, \lambda^{iA}, \lambda_A^{\bar{\imath}}, z^i\} \,, \tag{2.52}$$

where $i = 1, \ldots n_v$, n_v being the number of vector multiplets. The lower SU(2) index A on the gravitino (ψ_A, ψ^A) corresponds to positive chirality, while the corresponding upper index denotes negative chirality. The opposite convention is adopted for the gauginos $(\lambda^{iA}, \lambda_A^{\bar{\imath}})$. Finally, for the hyperinos fields $(\zeta_\alpha, \zeta^\alpha)$, lower and upper indices correspond to positive and negative chirality respectively.

In order to couple the scalar–tensor multiplet to the $\mathcal{N} = 2$ supergravity in the presence of vector multiplets, we use the superspace approach writing the curvatures of the various fields in superspace and solving the corresponding Bianchi identities. The curvatures are defined as follows [2, 15]

$$T^a \equiv \mathcal{D}V^a - \mathrm{i}\,\bar{\psi}_A \wedge \gamma^a \psi^A = 0 \,, \tag{2.53}$$

$$\rho_A \equiv d\psi_A - \frac{1}{4}\gamma_{ab}\,\omega^{ab}\wedge\psi_A + \frac{i}{2}\mathcal{Q}\wedge\psi_A + \omega_A{}^B\wedge\psi_B \equiv \nabla\psi_A\,, \tag{2.54}$$

$$\rho^A \equiv d\psi^A - \frac{1}{4}\gamma_{ab}\,\omega^{ab}\wedge\psi^A - \frac{i}{2}\mathcal{Q}\wedge\psi^A + \omega^A{}_B\wedge\psi^B \equiv \nabla\psi^A\,, \tag{2.55}$$

$$R^{ab} \equiv d\omega^{ab} - \omega^a{}_c\wedge\omega^{cb}\,. \tag{2.56}$$

In the vector multiplet sector the curvatures and covariant derivatives are:

$$\nabla z^i = dz^i\,, \tag{2.57}$$

$$\nabla\bar{z}^{\bar{\imath}} = d\bar{z}^{\bar{\imath}}\,, \tag{2.58}$$

$$\nabla\lambda^{iA} \equiv d\lambda^{iA} - \frac{1}{4}\gamma_{ab}\,\omega^{ab}\lambda^{iA} - \frac{i}{2}\mathcal{Q}\lambda^{iA} + \Gamma^i{}_j\lambda^{jA} + \omega^A{}_B\wedge\lambda^{iB}\,, \tag{2.59}$$

$$\nabla\bar{\lambda}^{\bar{\imath}}_A \equiv d\bar{\lambda}^{\bar{\imath}}_A - \frac{1}{4}\gamma_{ab}\,\omega^{ab}\bar{\lambda}^{\bar{\imath}}_A + \frac{i}{2}\mathcal{Q}\bar{\lambda}^{\bar{\imath}}_A + \Gamma^{\bar{\imath}}{}_{\bar{\jmath}}\bar{\lambda}^{\bar{\jmath}}_A + \omega_A{}^B\wedge\bar{\lambda}^{\bar{\imath}}_B\,, \tag{2.60}$$

$$F^\Lambda \equiv dA^\Lambda + \bar{L}^\Lambda\bar{\psi}_A\wedge\psi_B\epsilon^{AB} + L^\Lambda\bar{\psi}^A\wedge\psi^B\epsilon_{AB}\,. \tag{2.61}$$

In the scalar–tensor multiplet sector they are

$$H_I = dB_I + 2\,L_{I\,C}{}^A\,\bar{\psi}_A\gamma^a\psi^C V_a\,, \tag{2.62}$$

$$\nabla\zeta_\alpha = d\zeta_\alpha - \frac{1}{4}\omega^{ab}\gamma_{ab}\zeta_\alpha - \frac{i}{2}\,\mathcal{Q}\,\zeta_\alpha + \Delta_\alpha^\beta\zeta_\beta\,, \tag{2.63}$$

$$\nabla\zeta^\alpha = d\zeta^\alpha - \frac{1}{4}\omega^{ab}\gamma_{ab}\zeta^\alpha - \frac{i}{2}\mathcal{Q}\zeta^\alpha + \Delta_\beta^\alpha\,\zeta^\beta\,, \tag{2.64}$$

$$P^{A\alpha} = P_u^{A\alpha}dq^u\,, \tag{2.65}$$

where $\mathcal{Q}$ is the U(1) Kähler connection [2], Γ^i_j is the Christoffel connection one–form for the special Kähler manifold, L^Λ is part of the symplectic section (L^Λ, M_Λ) of the special manifold, ω^{AB} and $\Delta^{\alpha\beta}$ are respectively the SU(2) and Sp(2m, $\mathbb{R}$) connections (2.22), (2.24) on the $\mathcal{M}_n$ manifold and $L^A_{IC}(q)$ is a tensor to be determined by the Bianchi identities.

Differentiating equations (2.57)–(2.65) one obtains the Bianchi identities, whose super-space solutions are given by the following parameterizations (up to three fermion terms):

$$T^a = 0\,, \tag{2.66}$$

$$\rho_A = V^a V^b\rho_{abA} + \epsilon_{AB}\,T^-_{ab}\gamma^b\psi^B V^a - \frac{1}{2}h_{Ia}L^{IB}_A\psi_B V^a\,, \tag{2.67}$$

$$F^\Lambda = V^a V^b\,F^\Lambda_{ab} + \left(\mathrm{i}f^\Lambda_i\,\bar{\lambda}^{iA}\gamma_a\psi^B\epsilon_{AB} + \mathrm{i}\bar{f}^\Lambda_{\bar{\imath}}\,\bar{\lambda}^{\bar{\imath}}_A\gamma_a\psi_B\epsilon^{AB}\right)V^a\,, \tag{2.68}$$

$$\nabla\lambda^{iA} = V^a\nabla_a\lambda^{iA} + \mathrm{i}\,Z^i_a\,\gamma^a\psi^A + G^{-i}_{ab}\,\gamma^{ab}\psi_B\epsilon^{AB}\,, \tag{2.69}$$

$$H_I = H_{I\,abc}V^a V^b V^c + \frac{i}{2}\left(\bar{\psi}_A\gamma^{ab}\zeta_\alpha\,g^{A\alpha}_I\,V_a V_b - \bar{\psi}^A\gamma^{ab}\zeta^\alpha\,g_{I\,A\alpha}\,V_a V_b\right)\,, \tag{2.70}$$

$$\nabla\zeta_\alpha = V^a\nabla_a\zeta_\alpha + \mathrm{i}P_{aA\alpha}\gamma^a\psi^A + \mathrm{i}\,h_{Ia}\,g^I_{A\alpha}\,\gamma^a\psi^A\,, \tag{2.71}$$

$$\nabla z^i = Z_a^i V^a + \bar{\psi}_A \lambda^{iA} , \tag{2.72}$$

$$P^{A\alpha} = P_a^{A\alpha} V^a + \bar{\psi}^A \zeta^\alpha + \epsilon^{AB} \mathbb{C}^{\alpha\beta} \bar{\psi}_B \zeta_\beta . \tag{2.73}$$

We observe that the solution (2.66)–(2.69) of the Bianchi identities is the same as the ordinary $\mathcal{N} = 2$ supergravity coupled to vector multiplets, except for the extra term $h_{Ia} L_A^{IB} \psi_B V^a$ appearing in (2.67), where $h_{I\,a} = \epsilon_{abcd} H_I^{bcd}$. In particular, we have used the well known definitions for the self–dual dressed vector field strengths:

$$T_{ab}^- = \left(\mathcal{N} - \bar{\mathcal{N}} \right)_{\Lambda\Sigma} L^\Sigma F_{ab}^{-\Lambda} , \tag{2.74}$$

$$G_{ab}^{i-} = \frac{\mathrm{i}}{2} g^{i\bar{j}} f_{\bar{j}}^\Gamma \left(\mathcal{N} - \bar{\mathcal{N}} \right)_{\Gamma\Lambda} F_{ab}^{\Lambda-} . \tag{2.75}$$

and

$$f_i^\Lambda \equiv \nabla_i L^\Lambda , \tag{2.76}$$

where ∇_i is the Kähler covariant derivative. Furthermore the Bianchi identities imply that the tensors $g_I^{A\alpha}$, $P_a^{A\alpha} \equiv P_u^{A\alpha} \partial_a q^u$ and L_{AB}^I satisfy the following set of constraints

$$P_{A\alpha}^u P_v^{B\alpha} + P^{uB\alpha} P_{vA\alpha} = \delta_v^u \delta_A^B , \tag{2.77}$$

$$g^{AI\alpha} g_{JB\alpha} + g_{B\alpha}^I g_J^{A\alpha} = \delta_J^I \delta_B^A , \tag{2.78}$$

$$P_{aC\alpha} g^{IA\alpha} + P_a^{A\alpha} g_{C\alpha}^I = 0 , \tag{2.79}$$

$$P_{aC\alpha} g_I^{A\alpha} + P_a^{A\alpha} g_{IC\alpha} = 0 , \tag{2.80}$$

$$\nabla_a L_I^{CA} = P_{a\alpha}^{(C} g_I^{A)\alpha} , \tag{2.81}$$

$$g_J^{\alpha(A} g_{I\alpha}^{B)} = \mathrm{i}\epsilon_{xyz} L_I^y L_J^z \sigma_B^x{}^A . \tag{2.82}$$

It is important to note that the previous constraints on the tensors appearing in the scalar–tensor multiplet (coinciding with those found in reference [11]) are in complete agreement with the results obtained from the reduction of the quaternionic geometry of section 2.2. Indeed, equation (2.77) coincides with equations (2.13) and equation (2.80) coincides with (2.15) provided we identify

$$g_I^{A\alpha} \equiv \mathcal{U}_I^{A\alpha} . \tag{2.83}$$

For what concerns (2.79), we can also see that it can also be recast into the form of equation (2.15) upon identifying

$$g^{I\,A\alpha} \equiv M^{IJ} g_J^{A\alpha} = \mathcal{U}^{I\,A\alpha} + g^{uv} A_u^I P_v^{A\alpha} \tag{2.84}$$

With the identifications (2.83), (2.84) we also obtain (2.14) from equation (2.78). Equations (2.81), (2.82) instead coincide with (2.37) and (2.38) provided we identify

$$L_I^{AB} = -\frac{1}{2}\omega_I^{AB} \,. \tag{2.85}$$

Finally, for the benefit of the reader we write down the supersymmetry transformation laws of the fields on space–time as they follow immediately from the Bianchi identities solution (2.66)–(2.73):

$$\delta\,\psi_{A\mu} \;=\; \mathcal{D}_\mu\,\varepsilon_A + \epsilon_{AB}T^-_{\mu\nu}\gamma^\nu\varepsilon^B \tag{2.86}$$

$$\delta\,\lambda^{iA} \;=\; \mathrm{i}\,\nabla_\mu\,z^i\gamma^\mu\varepsilon^A + \epsilon^{AB}G^{-i}_{\mu\nu}\gamma^{\mu\nu}\varepsilon_B \tag{2.87}$$

$$\delta\,\zeta_\alpha \;=\; \mathrm{i}\,P_{u\,A\alpha}\,\partial_\mu\,q^u\,\gamma^\mu\varepsilon^A + \mathrm{i}h_{\mu I}g^I_{A\alpha}\gamma^\mu\varepsilon^A \tag{2.88}$$

$$\delta\,V^a_\mu \;=\; -\mathrm{i}\,\bar{\psi}_{A\mu}\,\gamma^a\,\varepsilon^A - \mathrm{i}\,\bar{\psi}^A_\mu\,\gamma^a\,\varepsilon_A \tag{2.89}$$

$$\delta\,A^\Lambda_\mu \;=\; 2\bar{L}^\Lambda\bar{\psi}_{A\mu}\varepsilon_B\epsilon^{AB} + 2L^\Lambda\bar{\psi}^A_\mu\varepsilon^B\epsilon_{AB} + \mathrm{i}\,f^\Lambda_i\,\bar{\lambda}^{iA}\gamma_\mu\varepsilon^B\,\epsilon_{AB} + \mathrm{i}\,\bar{f}^\Lambda_{i^\star}\,\bar{\lambda}^{i^\star}_A\gamma_\mu\varepsilon_B\,\epsilon^{AB} \tag{2.90}$$

$$\delta B_{\mu\nu\,I} \;=\; \frac{\mathrm{i}}{2}\left(\bar{\varepsilon}_A\gamma_{\mu\nu}\zeta_\alpha\,g_I^{A\alpha} - \bar{\varepsilon}^A\gamma_{\mu\nu}\zeta^\alpha\,g_{I\,A\alpha}\right) + 2L^A_{IC}\bar{\varepsilon}_A\gamma_{[\mu}\psi^C_{\nu]} + 2L^A_{IC}\bar{\psi}_{[\mu A}\gamma_{\nu]}\varepsilon^C \tag{2.91}$$

$$\delta\,z^i \;=\; \bar{\lambda}^{iA}\varepsilon_A \tag{2.92}$$

$$\delta\,z^{i^\star} \;=\; \bar{\lambda}^{i^\star}_A\varepsilon^A \tag{2.93}$$

$$\delta\,q^u \;=\; P^u_{\alpha A}\left(\bar{\zeta}^\alpha\varepsilon^A + C^{\alpha\beta}\epsilon^{AB}\bar{\zeta}_\beta\varepsilon_B\right) \tag{2.94}$$

3 The gauging

From the supersymmetry rules described in the previous section we have seen that the vector multiplet scalar manifold and its couplings have not been touched by the dualization procedure. This means that if the scalar manifold of the vector multiplets admits isometries, these can be gauged without any further restriction both for abelian and non–abelian gauge groups. On the other hand we have seen that the hypermultiplet scalar manifold now has been reduced and the resulting manifold contains the scalars q^u of the original hypermultiplets that have not been dualized. This means that care is needed in order to decide which isometries of the original quaternionic manifold can be gauged.

On general grounds, before the dualization procedure one could have, besides translations, isometries of the scalar manifolds acting non–trivially on the quaternions which we want to dualize into tensor fields. These isometries now become symmetries of the resulting manifold under which the tensor fields are charged. As a consequence one cannot make

these symmetries local without paying the price of introducing non–linear couplings of the tensor fields themselves [16]. In what follows we will therefore limit ourselves to the gauging of symmetries which commute with the translations of the dualized scalars. One should also remember that in order to gauge non–abelian groups, these symmetries should also be symmetries of the special Kähler manifold [2].

For these symmetries the gauging procedure is the standard one outlined in [2]. One adds to the (composite) connections appearing in the transformation laws of the charged fields the vector fields (which make this symmetry local) dressed with some function of the scalar fields. To be specific, the connections of the vector line bundle $\mathcal{Q}$, the connection of the vector tangent bundle Γ^i_j, the $SU(2)$ connection ω^x and the $Sp(2m)$ connection $\Delta^{\alpha\beta}$ are shifted as

$$\Gamma^i_j \quad \rightarrow \quad \Gamma^i_j + g\,A^\Lambda\,\partial_j k^i_\Lambda\,, \tag{3.1}$$

$$\mathcal{Q} \quad \rightarrow \quad \mathcal{Q} + g\,A^\Lambda\mathcal{P}^0_\Lambda\,, \tag{3.2}$$

$$\omega^x \quad \rightarrow \quad \omega^x + g\,A^\Lambda\mathcal{P}^x_\Lambda\,, \tag{3.3}$$

$$\Delta^{\alpha\beta} \quad \rightarrow \quad \Delta^{\alpha\beta} + g\,A^\Lambda\partial_u k^v_\Lambda\,\mathcal{U}^{u\alpha A}\mathcal{U}^\beta_{vA}\,. \tag{3.4}$$

These modifications obviously break the supersymmetry of the Lagrangian which can be restored by adding appropriate shifts to the supersymmetry rules of the Fermi fields as well as adding mass terms for the fermions in the Lagrangian at first order in g and a scalar potential which is of order g^2. Moreover the definitions (2.57), (2.58), (2.61) and (2.65) get modified to

$$\nabla z^i \quad = \quad dz^i + g\,A^\Lambda k^i_\Lambda\,, \tag{3.5}$$

$$\nabla \bar{z}^{\bar{\imath}} \quad = \quad d\bar{z}^{\bar{\imath}} + g\,A^\Lambda k^{\bar{\imath}}_\Lambda\,, \tag{3.6}$$

$$F^\Lambda \quad \equiv \quad dA^\Lambda + \frac{1}{2}\,g\,f^\Lambda_{\Gamma\Sigma}\,A^\Gamma \wedge A^\Sigma + \bar{L}^\Lambda\bar{\psi}_A \wedge \psi_B\epsilon^{AB} + L^\Lambda\bar{\psi}^A \wedge \psi^B\epsilon_{AB}\,, \tag{3.7}$$

$$P^{A\alpha} \quad = \quad P^{A\alpha}_u\left(dq^u + g\,A^\Lambda\,k^u_\Lambda\right)\,. \tag{3.8}$$

In addition to this standard electric gauging, for which the only constraint is the choice of generators commuting with the translations of the dualized scalars q^I, we can also introduce here mass terms for the tensor fields which will enter in the theory in a way that looks like a "magnetic gauging". We stress however, that we are not gauging the magnetic potentials and therefore the mass parameters can not be identified with magnetic charges since they are not related to the isometries of the reduced scalar manifold.

The appearance of tensor fields in the ungauged theory allows us for the introduction of explicit mass terms for these fields by redefining the (3.7) curvatures

$$\tilde{F}^\Lambda_{ab} = F^\Lambda_{ab} + m^{I\Lambda}B_{Iab}\,, \tag{3.9}$$

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where $m^{I\Lambda}$ are real constants. By doing so, the tensor fields appear naked in the Lagrangian and therefore cannot be trivially dualized back into scalar fields. Moreover this process will introduce explicit mass terms for B_I and indeed it can be viewed as the usual Stückelberg mechanism, where combinations of the gauge field strengths are absorbed in the definition of massive tensor fields. The introduction of these extra terms also breaks the supersymmetry of the Lagrangian. We will see in a while that we can again restore it by further shifting the supersymmetry transformations of the Fermi fields by terms of the first order in m and by modifying the scalar potential. It is actually interesting that these shifts will not simply produce m^2 terms in the potential but will also interfere with the electric shifts giving terms proportional to both couplings.

In the presence of the electric gauging and the mass parameters $m^{I\Lambda}$ the supersymmetry transformation laws of the fermionic fields (as they come from the closure of the gauged Bianchi identities) become:

$$\delta\,\psi_{A\mu} \;=\; \mathcal{D}_\mu\varepsilon_A + \epsilon_{AB}T^-_{\mu\nu}\varepsilon^B - \frac{1}{2}h_{\mu I}L^{IB}_A\varepsilon_B + \mathrm{i}\,S_{AB}\gamma^\mu\varepsilon^B\,, \tag{3.10}$$

$$\delta\,\lambda^{iA} \;=\; \mathrm{i}\,\nabla_\mu z^i\gamma^\mu\varepsilon^A + G^{-i}_{\mu\nu}\gamma^{\mu\nu}\epsilon^{AB}\varepsilon_B \,+\, W^{iAB}\varepsilon_B\,, \tag{3.11}$$

$$\delta\,\zeta_\alpha \;=\; \mathrm{i}\,P_{\mu A\alpha}\gamma^\mu\varepsilon^A + \mathrm{i}\,h_{I\mu}\,g^I_{A\alpha}\,\gamma^\mu\varepsilon^A \,+\, N^A_\alpha\,\varepsilon_A\,, \tag{3.12}$$

where the extra fermionic terms are:

$$S_{AB} \;=\; \frac{\mathrm{i}}{2}\,\sigma^x_{AB}\left(g\mathcal{P}^x_\Lambda L^\Lambda - L^x_I m^{I\Lambda}M_\Lambda\right)\,, \tag{3.13}$$

$$N^\alpha_A \;=\; -2\,g\,\mathcal{U}^\alpha_{Au}k^u_\Lambda L^\Lambda + g^\alpha_{IA}m^{I\Lambda}M_\Lambda\,, \tag{3.14}$$

$$W^{iAB} \;=\; -\mathrm{i}\,g\,\epsilon^{AB}g^{i\bar{j}}P_\Lambda f^\Lambda_{\bar{j}} + \mathrm{i}\,g^{i\bar{j}}\sigma^{AB}_x\left(g\mathcal{P}^x_\Lambda \bar{f}^\Lambda_{\bar{j}} - L^x_I m^{I\Lambda}h_{\Lambda\bar{j}}\right)\,, \tag{3.15}$$

where now the self–dual dressed vector field strengths (2.74), (2.75) appear combined with the tensor fields as in (3.9):

$$T^-_{ab} \;=\; \left(\mathcal{N} - \bar{\mathcal{N}}\right)_{\Lambda\Sigma}L^\Sigma \tilde{F}^{-\Lambda}_{ab}\,, \tag{3.16}$$

$$G^{i-}_{ab} \;=\; \frac{\mathrm{i}}{2}g^{i\bar{j}}\bar{f}^\Gamma_{\bar{j}}\left(\mathcal{N} - \bar{\mathcal{N}}\right)_{\Gamma\Lambda}\tilde{F}^{\Lambda-}_{ab}\,. \tag{3.17}$$

As expected, the dual sections of the vector scalar manifold appear in the magnetic parts of the above expressions

$$M_\Lambda = \mathcal{N}_{\Lambda\Sigma}L^\Sigma\,, \quad h_{\Lambda i} = \overline{\mathcal{N}}_{\Lambda\Sigma}f^\Sigma_i\,. \tag{3.18}$$

We notice that the electric part of the shifts has not changed from the standard one obtainable in the absence of tensor multiplets. In (3.14) there appear the Killing vectors of the isometries gauged of the scalar–tensor σ–model, which are the reduction of the isometries of

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the original quaternionic–Kähler manifold. The prepotentials associated to these isometries appear in (3.13). The geometrical interpretation of $\mathcal{P}_\Lambda^x$ cannot be the same of the standard theory, but the conditions following from the requirement of supersymmetry match (2.49), which are the relations following from the reduction of those existing between quaternionic Killing vectors and their prepotentials.

Consistency imposes a non–trivial constraint for the (3.13) and (3.14) shifts

$$4L_{IC}{}^{(A}\bar{S}^{B)C} = -N_\alpha^{(B}g_I^{A)\alpha}, \tag{3.19}$$

$$4L_{I(A}{}^C S_{B)C} = N^\alpha_{(A}g_{B)\alpha I}. \tag{3.20}$$

The electric part of these equations can be interpreted as the reduction of the geometric relations between quaternionic Killing vectors and prepotentials (2.47) on the external directions I, therefore matching (2.49). This can also be seen as a standard gradient flow equation [15]. The terms in the shifts proportional to $m^{I\Lambda}$ entering in equations (3.19), (3.20) identically solve them by using (2.38), where the identification (2.85) of the ω_I^x connection with L_I^x is done.

This last identity can also be interpreted as a gradient flow equation assuming that the $m^{I\Lambda}$ constants have the rôle of "magnetic" Killing vectors. In this way one can introduce "magnetic prepotentials" $\mathcal{Q}^{\Lambda x}$. Considering equation (2.47) for dual Killing vectors $\tilde{k}^{I\Lambda}$, for which we set $\tilde{k}^{u\Lambda} = 0$, one obtains

$$2\,\tilde{k}^{I\Lambda}\Omega_{IJ}^x \equiv -\nabla_J \mathcal{Q}^{x\Lambda} \equiv -\epsilon^{xyz}\omega_J^y \mathcal{Q}^{z\Lambda}, \tag{3.21}$$

where it was used that $\mathcal{Q}^{x\Lambda}$ should be independent on the scalars we dualized. We can now see that (3.19) and (3.20) are the same as (3.21) if we relate the Killing vector and the mass parameter as

$$\tilde{k}^{I\Lambda} = -\frac{1}{2}m^{I\Lambda}. \tag{3.22}$$

and define $\mathcal{Q}^{x\Lambda}$ as

$$\mathcal{Q}^{x\,\Lambda} = -\frac{1}{2}\omega_I^x m^{I\Lambda} = L_I^x m^{I\Lambda}. \tag{3.23}$$

By doing so almost all the terms in (3.13)–(3.15) can be rewritten in terms of symplectic invariants quantities. One can indeed introduce the symplectic vector

$$\mathcal{T}^x = \left\{\mathcal{Q}^{x\,\Lambda}, g\,\mathcal{P}_\Lambda^x\right\} \tag{3.24}$$

and rewrite (3.13)–(3.15) as contractions of this with the other symplectic vectors given by $V \equiv \{L^\Lambda, M_\Lambda\}$ and their derivatives

$$U_i = \nabla_i V \equiv \{f_i^\Lambda, h_{\Lambda i}\}. \tag{3.25}$$

Introducing also the symplectic vector

$$\mathcal{Z}_A^\alpha = \left\{ -g_{IA}^\alpha \, \tilde{k}^{I\Lambda} \, , \; g \, k_\Lambda^u \, \mathcal{U}_{uA}^\alpha \right\} = \left\{ 1/2 \, g_{IA}^\alpha \, m^{I\Lambda} \, , \; g \, k_\Lambda^u \, \mathcal{U}_{uA}^\alpha \right\} , \tag{3.26}$$

the shifts read

$$S_{AB} = \frac{i}{2} \sigma_{AB}^x < V, \mathcal{T}^x > , \tag{3.27}$$

$$N_A^\alpha = -2 < V, \mathcal{Z}_A^\alpha > , \tag{3.28}$$

$$W^{iAB} = -i \, g \, \epsilon^{AB} g^{i\bar{j}} P_\Lambda f_{\bar{j}}^\Lambda + i \, g^{i\bar{j}} \sigma_x^{AB} < U_{\bar{j}}, \mathcal{T}^x > , \tag{3.29}$$

where $<,>$ denotes the symplectic scalar product defined as follows:

$$\begin{pmatrix} a^\Lambda & b_\Lambda \end{pmatrix} \begin{pmatrix} 0 & \delta_\Lambda{}^\Sigma \\ -\delta^\Lambda{}_\Sigma & 0 \end{pmatrix} \begin{pmatrix} c^\Sigma \\ d_\Sigma \end{pmatrix} \tag{3.30}$$

This rewriting is especially useful in view of the construction of the scalar potential. Since the above expressions are all symplectic invariants, but for the antisymmetric piece in the gaugino's shift $W^{[AB]}$, also the corresponding contributions to the scalar potential will have the same properties.

So far we did not find any further constraint on the possible gauge group by adding magnetic charges, but we will see that this will not be the case anymore when considering the supersymmetry Ward identity of the scalar potential.

4   The potential

The scalar potential can be determined by a general Ward identity [15] of extended supergravities, which shows that it follows from squaring the fermion shifts. In the present case such identity reads

$$\delta_B^A \mathcal{V} = -12 \, \overline{S}^{CA} \, S_{CB} + g_{i\bar{j}} W^{iCA} \, W_{CB}^{\bar{j}} + 2 N_\alpha^A \, N_B^\alpha , \tag{4.1}$$

with S_{AB}, N_A^α and W^{AB} given by (3.13), (3.14) and (3.15) respectively.

As it is clear from the definition of the fermionic shifts, the right hand side of (4.1) contains both pieces proportional to δ_B^A as well as to $(\sigma^x)_B^A$. In order for the theory to be supersymmetric one has to prove that the parts which are $SU(2)$ Lie Algebra valued vanish identically. Since the various shifts contain expressions proportional to the electric coupling constant and others proportional to the "magnetic" one, their squares have three type of pieces which have to be set to zero separately. The first condition follows from the terms proportional to g^2 and it is the simple reduction of the quaternionic identity

$$\Omega_{uv}^x \, k_\Lambda^u k_\Sigma^v - \frac{1}{2} \epsilon^{xyz} \mathcal{P}_\Lambda^y \mathcal{P}_\Sigma^z + \frac{1}{2} f_{\Lambda\Sigma}^\Delta \, \mathcal{P}_\Delta^x = 0 . \tag{4.2}$$

Also the m^2 piece is identically satisfied. This can be seen from the fact that all contributions have the same form $\hat{\Omega}_{IJ\,A}{}^B\,m^{I\Lambda}m^{J\Sigma}M_\Lambda\bar{M}_\Sigma$ with the appropriate coefficients to make it vanish. The only non–trivial rewriting is the one involving the square of the gauginos, where using the identities of special geometry we find that

$$h_{\Lambda i}h_{\Sigma\bar{\jmath}}g^{i\bar{\jmath}} = -M_\Lambda\overline{M}_\Sigma + \frac{1}{2}(Im\mathcal{N})_{\Lambda\Sigma} \tag{4.3}$$

and the second part is identically vanishing since it is contracted with $\epsilon_{xyz}L_I^x L_J^y m^{\Lambda I}m^{\Sigma J}$. At this point we are left with the mixed contributions which give us a non–trivial constraint on the gauge group.

There are two different types of contributions to the Ward identity (4.1) which are proportional to both the electric and magnetic couplings and are triplets of $SU(2)$. The first one, is given by $\frac{i}{2}\mathcal{P}_{\Lambda AC}L_I^{CB}\,m^{I\Sigma}\,L^\Lambda\overline{M}_\Sigma+c.c.$ and is common to all the squares of the symplectic shifts. These again combine with the appropriate coefficients to give zero. In addition, from the gauginos one has a contribution which is an interference term of the gauging of the vector multiplet isometries and the magnetic gauging. This reads

$$g^{i\bar{\jmath}}W_{iAC}W_{\bar{\jmath}}^{CB} \sim \mathrm{i}k_\Lambda^i\,h_{\Sigma i}L_{IA}{}^C m^{I\Sigma}L^\Lambda + c.c. \tag{4.4}$$

and it should vanish by itself. The group–theoretic meaning of such equation becomes clear by using the known identity of special geometry [2]

$$k_\Lambda^i f_i^\Gamma = f_{\Lambda\Pi}^\Gamma\overline{L}^\Pi + \mathrm{i}\mathcal{P}_\Lambda^0\,V\,. \tag{4.5}$$

Once this equation is used in (4.4) the constraint becomes

$$f_{\Lambda\Sigma}^\Delta\,(Im\mathcal{N})_{\Delta\Pi}\,m^{I\Pi} = f_{\Lambda\Sigma}^\Delta\tilde{m}_\Delta^I = 0\,, \tag{4.6}$$

where

$$\tilde{m}_\Delta^I \equiv (Im\mathcal{N})_{\Delta\Pi}\,m^{I\Pi}\,. \tag{4.7}$$

Note that this equation means that $\tilde{m}_\Delta^I$ are the coordinates of a Lie algebra element of the gauge group commuting with all the generators, in other words a non trivial element of the center $\mathbb{Z}(\mathbb{G})$. Such a constraint follows only for non–abelian gaugings because that is the only case when one is forced to gauge explicitly the isometries of the vector scalar manifold introducing the vector manifold Killing vectors k_Λ^i. In our case, this condition is certainly satisfied since we assumed from the beginning that the isometries that can be gauged electrically are only those that commute with the translational isometries of the dualized scalars. As it should be obvious at this stage, the "magnetic Killing vectors" $m^{I\Lambda}$ are precisely associated to such translational symmetries.

Once one has verified that the Ward identity (4.1) of supersymmetry is satisfied for the $SU(2)$ Lie Algebra valued pieces one can finally read the potential

$$
\begin{aligned}
\mathcal{V} =\ & g^2 \left\{ g_{i\bar{\jmath}} k^i_\Lambda k^{\bar{\jmath}}_\Sigma \overline{L}^\Lambda L^\Sigma + 4(g_{uv} + h_{Iu} M^{IJ} h_{Jv}) k^u_\Lambda k^v_\Sigma \overline{L}^\Lambda L^\Sigma + \left(U^{\Lambda\Sigma} - 3\overline{L}^\Lambda L^\Sigma \right) \mathcal{P}^x_\Lambda \mathcal{P}^x_\Sigma \right\} \\
& + g \left[g^{i\bar{\jmath}} \left(f^\Lambda_i h_{\bar{\jmath}\Sigma} + f^\Lambda_{\bar{\jmath}} h_{i\Sigma} \right) - 3 \left(\overline{M}_\Lambda L^\Sigma + M_\Lambda \overline{L}^\Sigma \right) \right] \mathcal{P}^x_\Sigma \mathcal{Q}^{x\Lambda} \\
& - 2g\, h_{uI} \left(\overline{L}^\Lambda M_\Sigma + L^\Lambda \overline{M}_\Sigma \right) k^u_\Lambda\, m^{I\Sigma} \\
& + M_{IJ} m^{I\Lambda} m^{J\Sigma} M_\Lambda \overline{M}_\Sigma + \left(g^{i\bar{\jmath}} h_{i\Lambda} h_{\bar{\jmath}\Sigma} - 3\overline{M}_\Lambda M_\Sigma \right) \mathcal{Q}^{x\Lambda} \mathcal{Q}^{x\Sigma} .
\end{aligned}
\tag{4.8}
$$

In the absence of "magnetic charges" only the first line of (4.8) is different from zero and corresponds to the standard $\mathcal{N} = 2$ potential where $g_{uv} + h_{Iu} M^{IJ} h_{Jv}$ has to be identified with the metric of the quaternionic–Kähler manifold of the hypermultiplet sector.

It should be noted that the full potential (4.8) is symplectic invariant, being the square of symplectic invariant quantities, except for the first term given by the square of the Killing vectors of the special Kähler geometry. One can rewrite $\mathcal{V}$ by using (3.24) and (3.26). In doing so we find that the potential is a sum of four distinct pieces

$$
\mathcal{V} = g^2\, g_{i\bar{\jmath}} k^i_\Lambda k^{\bar{\jmath}}_\Sigma \overline{L}^\Lambda L^\Sigma + (\mathcal{T}^x)^T \mathcal{M}_S \mathcal{T}^x + 4 \left[(\mathcal{Z}^\alpha_A)^T \mathcal{M}_N \mathcal{Z}^A_\alpha - (\mathcal{T}^x)^T \mathcal{M}_N \mathcal{T}^x \right] .
\tag{4.9}
$$

where the scalar product of the last three terms, coming from the square of symplectic invariant products, have been rewritten as ordinary (orthogonal) matrix product. This implies that the matrices $\mathcal{M}_S$ and $\mathcal{M}_N$

$$
\mathcal{M}_S = -\frac{1}{2} \begin{pmatrix} \mathcal{I}_{\Lambda\Sigma} + (\mathcal{R}\mathcal{I}^{-1}\mathcal{R})_{\Lambda\Sigma} & -(\mathcal{R}\mathcal{I}^{-1})_\Lambda{}^\Sigma \\ -(\mathcal{I}^{-1}\mathcal{R})^\Lambda{}_\Sigma & \mathcal{I}^{-1\,\Lambda\Sigma} \end{pmatrix}
\tag{4.10}
$$

and

$$
\mathcal{M}_N = \begin{pmatrix} \overline{M}_\Lambda M_\Sigma & -\overline{M}_\Lambda L^\Sigma \\ -\overline{L}^\Lambda M_\Sigma & \overline{L}^\Lambda L^\Sigma \end{pmatrix} ,
\tag{4.11}
$$

where we introduced $\mathcal{I} \equiv Im\mathcal{N}$ and $\mathcal{R} \equiv Re\mathcal{N}$ to make the notation compact, need not to be symplectic (in spite of this $\mathcal{M}_S$ turns out to be symplectic).

Since the term which explicitly breaks symplectic invariance appears only when one gauges non–abelian groups any abelian electric gauging leads to a symplectic invariant potential. It is interesting to point out that precisely this type of gauged supergravities with tensors appear naturally in compactifications of type II string theories in the presence of non–trivial fluxes for the Ramond–Ramond and Neveu–Schwarz three–form fields. For the special case of Calabi–Yau compactifications, though, we do not simply find that $g_{i\bar{\jmath}} k^i_\Lambda k^{\bar{\jmath}}_\Sigma \overline{L}^\Lambda L^\Sigma = 0$ as expected, but another important cancellation happens. For abelian isometries the definition of the prepotentials (2.47) simplifies [4, 15, 6] to

$$
\mathcal{P}^x_\Lambda = \omega^x_{\hat{u}} k^{\hat{u}}_\Lambda .
\tag{4.12}
$$

Once this is projected on the reduced scalar manifold following from Calabi–Yau compactifications and we take into account the analogous equation (3.23), we obtain that

$$(\mathcal{Z}_A^\alpha)^T \mathcal{M}_N \, \mathcal{Z}_\alpha^A = (\mathcal{T}^x)^T \mathcal{M}_N \mathcal{T}^x \,, \tag{4.13}$$

which eventually implies that the scalar potential becomes positive–definite and explicitly symplectic invariant

$$\mathcal{V} = (\mathcal{T}^x)^T \mathcal{M}_S \mathcal{T}^x \,. \tag{4.14}$$

We stress here that this result follows only for Calabi–Yau compactifications and a standard choice of the symplectic sections V, that is one for which the prepotential function of Special Geometry exists. In the generic case one cannot conclude that (4.13) holds, though usually analogous cancellations between positive and negative terms of the potential also appear for gaugings of translational isometries, again leading to positive semi–definite potentials [17, 8, 9].

Let us now establish the relation between our work and the standard formulation of [2]. At the level of the ungauged theory the two formulations are simply related by a dualization procedure, that is a Legendre transformation. Once the theory is deformed by the gauging, the dualization of the tensors into scalars cannot be performed anymore. For Abelian gaugings, though, we have seen that the scalar potential exhibits an explicit symplectic invariance and one can use this invariance to put the potential in the standard form of [2]. In order to do so, one would like to remove all the dependence on the "magnetic charges" $m^{I\Lambda}$, i.e. set $\mathcal{Q}^x = 0$. Since $\mathcal{T}^x$ is in general a local function of the coordinates we cannot rotate it to a configuration where $\mathcal{Q}^x = 0$ by a constant symplectic matrix. For special cases, though, if the number of abelian factors is the same as the number of tensors, one can find that the symplectic vector $\mathcal{T}^x$ can be written as an overall function of the scalars multiplying a vector given by constant electric and magnetic charges so that $\mathcal{T}^x \sim \{e_\Lambda^I, m^{I\Lambda}\}$, where the extra I index on the electric charges follows from assigning different charges to the different abelian factors gauged. However, since the "magnetic charges" have an extra index I, it is impossible to put all of them to zero by a symplectic rotation unless the $\{e_\Lambda^I, m^{I\Lambda}\}$ are parallel vectors for all I's, or in the case where there is just one tensor, $I = 1$. The standard formulation can thus be retrieved only in such cases. For non Abelian gaugings we cannot in any case reduce the theory to the standard formulation with only electric charges, due to the presence of the non–symplectic invariant part.

The case of Abelian gaugings with parallel charge vectors is realized in Calabi–Yau compactifications with two electric and two magnetic charges coming from the Ramond–Ramond and Neveu–Schwarz 2–forms of the Type IIB theory. In this case the tadpole cancellation

condition implies that $e_\Lambda^1 m^{2\Lambda} - e_\Lambda^2 m^{1\Lambda} = 0$ (local case) [5]. Therefore we can choose a symplectic rotation such that the magnetic charges are set to zero and the symplectic vector (3.24) contains only the electric prepotentials. This also implies that the potential (4.8) becomes of the form given in [2, 15] for Abelian gaugings of the quaternionic manifold. In detail, it can be shown to be

$$\mathcal{V} = (\mathcal{T}'^x)^T \mathcal{M}_S \mathcal{T}'^x = -\frac{1}{2}\mathcal{I}^{-1\Lambda\Sigma}\mathcal{P}_\Lambda'^x \mathcal{P}_\Sigma'^x \, , \tag{4.15}$$

which is of the form presented in [5, 6, 7]. Note, however, that, as shown in reference [18], for orientifold Calabi–Yau compactifications, the parallelism condition cannot be imposed if we want to obtain a zero potential at the extrema.

We conclude that, except for the afore mentioned particular cases, the deformed theory in the presence of tensor multiplets cannot be simply related to the one without them. This seems to indicate that for such gaugings we find new genuine deformations containing tensor fields which are inequivalent to the formulation presented in [2]. It would be very interesting to find a derivation of these new supergravities as consistent effective low–energy formulations of string or M–theory, possibly in the presence of fluxes.

A   Examples

In this appendix we give two examples of how to retrieve the geometry of the scalar manifold after dualization in the simple case of the quaternionic manifolds $SU(2,1)/SU(2) \times U(1)$ and $SO(4,1)/SO(4)$. Furthermore in the third example we justify our assertion that dualization of a compact coordinate gives generally a singular Lagrangian; this is done for the σ–model $SU(1,1)/U(1)$.

Let us first discuss the dualization procedure for two simple quaternionic manifolds, namely $SU(2,1)/U(2)$ and $SO(4,1)/SO(4)$.

Example 1: $SU(2,1)/U(2)$

In accordance to our discussion in the text (2.11), we decompose the algebra of $SU(2, 1)$ where $\mathfrak{h} = \mathfrak{su}(2) + \mathfrak{u}(1)$, $\mathbb{D} = H$, $\mathbb{N} = \{G, T_1, T_2\}$, where H is the non compact Cartan generator of $\mathfrak{su}(2, 1)$ and $\{G, T_1, T_2\}$ are the nilpotent generators corresponding to the solvable algebra generating the coset manifold. The commutation relations of the solvable algebra are:

$$[H, G] = G, \qquad [H, T_i] = \frac{1}{2} T_i, \quad i = 1, 2, \tag{A.1}$$

$$[T_1, T_2] = G, \qquad [G, T_i] = 0, \tag{A.2}$$

We choose as maximal abelian ideal the set of generators $\{G, T_1\}$. The coset representative will be defined as

$$L = e^{\sigma G} \, e^{\varphi_1 T_1} \, e^{\varphi_2 T_2} \, e^{\phi H}, \tag{A.3}$$

where the fields $\{\sigma, \varphi_1, \varphi_2, \phi\}$ are the scalar fields of the σ–model, the subset $\{\sigma, \varphi_1\}$ being associated to the Peccei–Quinn symmetries.

Let us first compute the metric of the quaternionic manifold $SU(2, 1)/SU(2) \times U(1)$; we have

$$L^{-1}dL = Hd\phi + \left(e^{-\frac{\phi}{2}} T_1 + e^{-\phi}\varphi_2 G \right) d\varphi_1 \; + e^{-\frac{\phi}{2}} T_2 d\varphi_2 + e^{-\phi} G d\sigma. \tag{A.4}$$

Note that the nilpotent generators $\{G, T_1, T_2\}$ have a non compact and a compact part. If we denote with an hat the non compact part and normalize the traces to a Kronecker delta, namely:

$$Tr(\hat{G}H) = Tr(\hat{T}_i G) = Tr(\hat{T}_i H) = 0, \qquad Tr(\hat{T}_i^2) = Tr(\hat{g}^2) = Tr(\hat{H}^2) = 1, \tag{A.5}$$

the metric is easily computed and we find:

$$ds^2 = \mathrm{Tr} \left(L^{-1}dL_{G/H} \right)^2 = e^{-\phi} \left(d\varphi_1^2 + d\varphi_2^2 \right) + d\phi^2 + e^{-2\phi} \left(d\sigma + \varphi_2 \, d\varphi_1 \right)^2. \tag{A.6}$$

It is very easy at this point to recover the scalar manifolds of the σ–models associated to dualization of the coordinates σ and φ_1. It is sufficient to kill the associated generators $\{G, T_1\}$ in equation (A.4), so that the metric of the resulting manifold is obtained simply by setting to zero the coordinates σ and φ_1 and their differentials in equation (A.6). We thus obtain that the metric of the σ– model of the double tensor multiplet is given in this case by:

$$ds^2 = e^{-\phi}d\varphi_2^2 + d\phi^2, \tag{A.7}$$

which is easily seen to correspond to the coset manifold $SO(2, 1)/SO(2)$.

If we instead dualize just one coordinate, say σ, by the same procedure we find that the σ– model of the single tensor multiplet has the metric:

$$ds^2 = e^{-\phi} \left(d\varphi_1^2 + d\varphi_2^2 \right) + d\phi^2, \tag{A.8}$$

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which corresponds to the manifold $SO(3,1)/SO(3)$.

Example 2: $SO(4,1)/SO(4)$

For the manifold $SO(4,1)/SO(4)$ we proceed in an analogous way. We decompose the algebra $\mathfrak{g} = \mathfrak{so}(4,1)$ in terms of $\mathfrak{h}$, $\mathbb{D} = H$ and $\mathbb{N} = \{T_1, T_2, T_3\}$, where again H is a non compact Cartan generator of $\mathfrak{so}(4,1)$ and $\mathbb{N}$ describe the nilpotent ones. Explicitly, the $\mathfrak{so}(4,1)$ algebra is generated by T^{ij} with $i, j = 0, \ldots, 4$, satisfying

$$[T^{ij}, T_{kl}] = -4\, \delta^{[i}{}_{[k} T^{j]}{}_{l]}\,, \tag{A.9}$$

where indices are lowered by $\eta = \text{diag}\{-+++\}$. A solvable decomposition follows then by defining

$$H = T^{04}\,, \qquad T_i = T^{i0} - T^{i4}\,, \quad i = 1, 2, 3\,. \tag{A.10}$$

The commutation relations of the solvable generators are

$$[H, T_i] = T_i\,, \qquad [T_i, T_j] = 0\,, \tag{A.11}$$

for any i. The coset representative is now chosen as

$$L = \left(\Pi_i\, e^{\varphi_i T_i}\right) e^{\rho H}\,, \tag{A.12}$$

where the fields $\{\rho, \varphi_i\}$ are the scalar fields of the σ–model and the x_i are associated with translational isometries. Proceeding as in the previous example we obtain for the metric of $SO(4,1)/SO(4)$

$$ds^2 = \text{Tr}\left(L^{-1} dL_{G/H}\right)^2 = d\rho^2 + e^{2\rho}\left(d\varphi_1^2 + d\varphi_2^2 + d\varphi_3^2\right)\,. \tag{A.13}$$

It is now easy to see that the resulting manifolds after dualization of any of the φ_i scalars yields $SO(3,1)/SO(3)$ and that by the same procedure one can also obtain a double tensor multiplet with scalar manifold $SO(2,1)/SO(2)$ and even a triple–tensor multiplet with scalar manifold $SO(1,1)$. More in detail, after an appropriate identification of ρ with ϕ, by dualizing φ_3 one obtains (A.8) and by dualizing both φ_3 and φ_1 one gets (A.7).

We stress that although the metric for the double tensor multiplet is always (A.7) one can dualize the remaining scalar φ_2 only for the theory coming from $SO(4,1)/SO(4)$ and not for the universal hypermultiplet. This happens because the metric of the kinetic term of the double tensor multiplet coming from the $SU(2,1)/U(2)$ manifold contains explicitly φ_2 whereas the kinetic terms of the tensors for the theory following from $SO(4,1)/SO(4)$ depend only on ϕ.

Example 3: $SU(1,1)/U(1)$

In this last example we give an example of dualization of a compact coordinate giving rise to a singular Lagrangian, taking as toy model SU(1, 1)/U(1).

Let us consider the σ–model with metric, on the unit disk,

$$ds^2 = \frac{dzd\bar{z}}{(1 - z\bar{z})^2}\,, \tag{A.14}$$

or, introducing polar coordinates,

$$ds^2 = \frac{1}{(1 - \rho^2)^2}(d\rho^2 + \rho^2 d\theta^2)\,. \tag{A.15}$$

Suppose we dualize the compact coordinate θ. Introducing Lagrangian multipliers H_μ we have the following σ–model Lagrangian

$$\mathcal{L} = \frac{1}{(1 - \rho^2)^2}\partial_\mu\rho\partial_\nu\rho g^{\mu\nu} + aH_\mu H^\mu + H^\mu\partial_\mu\theta\,. \tag{A.16}$$

Varying θ we find $\partial_\mu H^\mu = 0$, that is H_μ is the dual of the 3–form. Varying H_μ one finds $H_\mu = -\frac{1}{2a}\partial_\mu\theta$. Substituting in (A.16) and comparing with (A.15) fixes the value of a to be $a = -\frac{(1-\rho^2)^2}{\rho^2}$. The dualized Lagrangian is then found by using $\partial_\mu\theta = -2aH_\mu$ in (A.16) and one finally obtains

$$\mathcal{L}^{Dual} = \frac{\partial_\mu\rho\partial^\mu\rho}{(1 - \rho^2)^2} + \frac{(1 - \rho^2)^2}{\rho^2}H_\mu H^\mu\,. \tag{A.17}$$

Equation (A.17) shows explicitly that the Lagrangian is singular in $\rho = 0$ that is at the origin of the coset manifold. If we instead set $z = \frac{i-S}{i+S}$ so that the unit disk is conformally mapped into the upper half plane the corresponding metric takes the form

$$ds^2 = \frac{dSd\bar{S}}{(ImS)^2}\,, \tag{A.18}$$

where we set $S = ie^\phi + C$. The same procedure used before gives now, as dualization of the corresponding σ–model, the following dual Lagrangian

$$\mathcal{L}^{Dual} = \partial_\mu\phi\partial^\mu\phi + \frac{1}{4}e^{2\phi}H_\mu H^\mu\,, \tag{A.19}$$

where no singularity appears. Indeed our parametrization with the S field corresponds exactly to performing the dualization of the non–compact generator of the solvable Lie algebra of SU(1, 1)/U(1)

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